

AP Stat- Ch. 15 (part 2) **TREE DIAGRAMS (Bayes' Rule)**

**EXAMPLE:** An airline offers discounted tickets to customers who buy tickets early (more than 30 days ahead of time). The company has noticed that 60% of its customers take advantage of the "early-bird" fares, while 25% purchase regular fares, and 15% are "last-minute" customers (less than 48 hours before the flight). They have also figured out that the no-show rate among "early-bird" purchasers is 30%, and only 8% among regular fare customers, and 2% among last-minute customers.

Create a tree diagram of this situation:

$$P(E) = 0.60$$

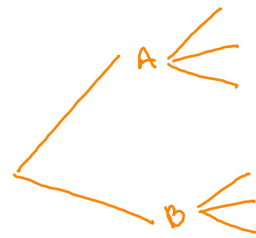
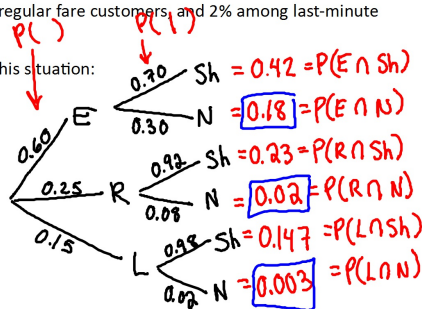
$$P(R) = 0.25$$

$$P(L) = 0.15$$

$$P(N|E) = 0.30$$

$$P(N|R) = 0.08$$

$$P(N|L) = 0.02$$



Easy questions:

- 1) What is the probability of being a no-show (overall)?

$$P(N) = 0.203$$

- 2) What is the probability of being a no-show AND a last minute customer?

$$P(N \cap L) = 0.003$$

- 3) What is the probability of being a no-show AND an early-bird customer?

$$P(N \cap E) = 0.18$$

Harder Questions:

1. Given that you have a no-show in a certain seat, what is the probability of that person being a last-minute customer?

$$P(L|N) = \frac{P(L \cap N)}{P(N)} = \frac{0.003}{0.203} = 0.0148$$

2. Given that you have a no-show in a certain seat, what is the probability of that person being a regular-fare customer?

$$P(R|N) = \frac{P(R \cap N)}{P(N)} = \frac{0.02}{0.203} = 0.0985$$

3. Given that you have a person show up for a certain seat, what is the probability of that person being an early-bird customer?

$$P(E|Sh) = \frac{0.42}{0.797} = 0.527$$

Example #1

- A cancer clinic gives free cancer test (hypothetically)
- It is known that 2% of the people that come into the clinic have cancer (hypothetically)
- It is known the test comes up positive in 98% of people with cancer
- It is known the test comes up positive in 3% of people without cancer

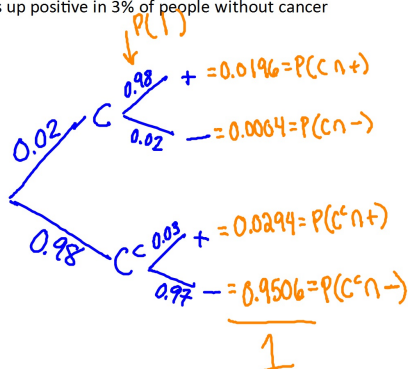
Create the tree diagram:

~~What is the probability that someone tests positive given that they have cancer?~~

$$P(C) = 0.02$$

$$P(+|C) = 0.98$$

$$P(+|C^c) = 0.03$$



Answer the following questions:

- 1- What is the probability that someone tests positive given that they have cancer?

$$P(+|C) = 0.98$$

- 2- What is the probability that someone tests positive given that they don't have cancer?

$$P(+|C^c) = 0.03$$

- 3- What is the probability that someone tests negative given that they have cancer?

$$P(-|C) = 0.02$$

- 4- What is the probability that someone tests negative given that they don't have cancer?

$$P(-|C^c) = 0.97$$

- 5- What is the probability that someone tests positive? Negative?

$$P(+) = 0.049 \quad P(-) = 0.951$$

6- What is the probability that someone has cancer **given that** they test positive?  
(This is called the accuracy of the test)

$$P(C|+) = \frac{P(C \cap +)}{P(+)} = \frac{0.0196}{0.049} = 0.40$$

7- What is the probability that someone doesn't have cancer **given that** they test positive?  
(this is called a false positive)

$$P(C^c|+) = \frac{P(C^c \cap +)}{P(+)} = \frac{0.0294}{0.049} = 0.60$$

8- What is the probability that someone has cancer **given that** they test negative?

$$P(C|N) = \frac{P(C \cap N)}{P(N)} = \frac{0.0004}{0.951} = 0.000421$$

$4.21 \times 10^{-4}$

#### Example #2:

- There are 2 textbook making companies, A and B
- It is known that 1% of company A's books are defective
- It is known that 2% of company B's books are defective
- CB South gets 38% of its books from company A and the rest from company B

#### Questions:

- 1- What is the probability that a book is NOT defective?
- 2- If a book is not defective, what's the probability that it came from company B?
- 3- If we open a book and it **IS** defective, what's the probability that is from company A? Company B?

$$\begin{aligned} P(D|A) &= 0.01 \\ P(D|B) &= 0.02 \\ P(A) &= 0.38 \\ P(B) &= 0.62 \end{aligned}$$

$P(D|A) = 0.01$   
 $P(D|B) = 0.02$   
 $P(A) = 0.38$   
 $P(B) = 0.62$

①  $P(D^c) = 0.9838$   
 ②  $P(B|D^c) = \frac{P(B \cap D^c)}{P(D^c)} = \frac{0.6076}{0.9838} = 0.6176$   
 ③  $P(A|D) = \frac{P(A \cap D)}{P(D)} = \frac{0.0038}{0.0162} = 0.2346$   
 $P(B|D) = \frac{P(B \cap D)}{P(D)} = \frac{0.0124}{0.0162} = 0.7654$

#### ANSWERS:

1-  $P(D^c) = 0.9838$

2-  $P(B|D^c) = 0.6176$

3-  $P(A|D) = 0.235$

$P(B|D) = 0.7654$

$$\frac{P(A \cap D)}{P(D)}$$

$$\frac{P(B \cap D)}{P(D)}$$

#### TRY THE TWO PROBLEMS ON THE NEXT PAGE and THE WORKSHEET AFTER THAT

1. A VCR manufacturer receives 70% of its parts from factory A and the rest from factory B. Suppose that 3% of the output from A are defective, while only 2% of the output from B are defective.

- What is the probability that a received part is defective?
- If a randomly chosen part is defective, what is the probability that it came from factor A? From factory B?

2. A particular football team is known to run 30% of its plays to the left and 70% to the right. A linebacker on an opposing team notes that the right guard shifts his stance most of the time (80%) when plays go to the right and that he uses a balanced stance the rest of the time. When plays go left, the guard takes a balanced stance 90% of the time and the shift stance the remaining 10%. What is the probability that a play will go to the left if the guard is balanced?

1. Ninety percent of the insulators produced by a company are satisfactory. The company hires an inspector. The inspector checks all the insulators and correctly classifies an item 95% of the time. Items classified as good are shipped and those classified defective are scrapped.

- What percentage of items will be classified as good?
- What percentage of items *shipped* can be expected to be good?

2. Suppose that three branches of a local bank average, 120, 180, and 100 clients per day, respectively. Suppose further that the probabilities that a client will transact business involving more than \$100 during a visit are .5, .6, .7, respectively. A client is chosen at random.

- What is the probability that the client will transact business involving over \$100?
- What is the probability that the client went to the first branch given that she transacted business involving over \$100?

